ISRAEL JOURNAL OF MATHEMATICS, Vol. 66, Nos. 1-3, 1989

THE WORK OF ALEXANDER ZABRODSKY 1936–1986

BY
JOHN HARPER
Department of Mathematics, University of Rochester, Rochester, NY 14627, USA

Alex Zabrodsky received the Master of Science degree from the Technion — Israel Institute of Technology at Haifa. Two publications appeared.

In *Covering spaces of paracompact spaces* he obtains the following result: *Suppose* $\tilde{X} \xrightarrow{f} X$ *is a covering of a metrizable locally connected space* X. *Then there exist metrics* $\tilde{\rho}$ *for* $\tilde{X}$ *and* ρ *for* X *which induce the topologies for* $\tilde{X}$ *and* X, *and such that the inverse image under* f *of any unit ball in* X *is a disjoint union of unit balls in* $\tilde{X}$, *each mapped isometrically by* f.

Following study in Israel, Alex went to Princeton, where he wrote his Ph.D. thesis under the supervision of Norman Steenrod and William Browder. This thesis, *On the structure of the cohomology of H-spaces*, was completed in 1967. An improved and generalized version appeared as *Implications in the cohomology of H-spaces* [5].

As the titles of his doctoral work suggest, Zabrodsky's work at this period concerned torsion in the homology of H-spaces. The technical point of departure was a process called "infinite implications", which had been initiated by W. Browder. Following Browder, we say that an element x in a Hopf algebra H has 1-*implication* provided either that $x^p \neq 0$ or that there is an element $\bar{x}$ in the dual Hopf algebra H^* such that $\langle x, \bar{x}\rangle \neq 0$ and $\bar{x}^p \neq 0$. Here we assume that H is of finite type over a field of characteristic p. Now if H is the mod p cohomology of an H-space and x has dimension $2n$, then $x^p = \mathcal{P}^n x$, and half the implication process is detected by a primary operation. Zabrodsky's contribution was to show how the other half of the implication process could be detected by a secondary cohomology operation.

Received May 2, 1988

He constructs certain two-stage Postnikov systems which are universal examples for p-th powers in homology. Before describing these examples, we briefly recount the theory of secondary cohomology operations and their universal examples.

To fix ideas, consider a relation $\Sigma b_i a_i = 0$ in the mod p Steenrod algebra $\mathscr{A}$ (or in $\mathscr{A}/B(n)$ when unstable considerations are paramount, where $B(n)$ is the left ideal annihilating n-dimensional cohomology classes). Associated with this data is a function

$$\varphi : \bigcap \ker a_i \rightarrow H^*/\bigoplus \mathrm{im}\, b_i.$$

A universal example for φ is a space E and two classes u, e in the cohomology of E such that for any $x \in H^*(X)$ in the domain of φ

$$\varphi(x) = \{ f^*(e) \mid f: X \rightarrow E \text{ such that } f^*(u) = x \}.$$

A universal example (for classes in dimension n) for φ is built as follows. Form the pull-back E,

$$\begin{array}{ccc} E & \longrightarrow & \mathscr{L}K_0 \\ \downarrow & & \downarrow \\ K(Z/pZ, n) & \xrightarrow{\alpha} & K_0 \end{array}$$

where

$$\alpha : K(Z/pZ, n) \rightarrow \prod_i K(Z/pZ, n_i) \doteq K_0$$

is given, up to homotopy, by

$$\alpha^*(\iota_{n_i}) = a_i \iota_n, \qquad n_i = n + \deg a_i,$$

and $\mathscr{L}K_0$ is the space of based paths in K_0.

Consider a map

$$\beta : K_0 \rightarrow K(Z/pZ, m), \qquad m = \deg a_i + \deg b_i$$

given, up to homotopy, by

$$\beta^* \iota_m = b_i \iota_{n_i}.$$

The relation $\Sigma b_i a_i \equiv 0 \bmod B(n)$ implies that the composition $\beta \circ \alpha$ is null-homotopic. Any null-homotopy

$$L : K(Z/pZ, n) \rightarrow \mathscr{L}K(Z/pZ, m)$$

can be used to define a map

$$\varepsilon_L : E \to \Omega K(Z/pZ, m)$$

by the formula

$$\varepsilon_L(x, w) = \mathscr{L}\beta \circ w - L(x).$$

Then after identifying $\Omega K(Z/pZ, m)$ with $K(Z/pZ, m-1)$, we have

$$e = \varepsilon_L^*(\imath_{m-1}).$$

To describe φ, we let x be an n-dimensional class satisfying $a_i x = 0$. Let $f: X \to K(Z/pZ, n)$ satisfy $f^*\imath_n = x$, then φ is given by

$$\varphi(x) = \{\tilde{f}^*(e) \mid \tilde{f}: X \to E \text{ lifts } f\},$$

and this set can be identified with a coset of the sum of the images of b_i. Furthermore, the space E has a natural loop space structure.

We can now describe Zabrodsky's theorem connecting implications with secondary operations. Consider any relation

$$\beta\mathscr{P}^n = \Sigma b_i a_i.$$

Since $\beta\mathscr{P}^n$ annihilates $2n$-dimensional classes, we have a universal example $E \xrightarrow{\pi} K(Z/pZ, 2n)$ and a class

$$e \in H^{2np}(E; Z/pZ).$$

Then Zabrodsky proves the following *implication theorem*,

$$\langle e, \bar{\imath}_{2n}^p \rangle \neq 0$$

where $\bar{\imath}_{2n} \in H_{2n}(E; Z/pZ)$ satisfies $\langle \imath_{2n}, \pi_* \bar{\imath}_{2n} \rangle \neq 0$. In other words, the image of the fundamental class in $H^*(E; Z/pZ)$ has 1-implication.

Thus both halves of the implication process are connected with the action of the Steenrod algebra. Now Browder's work had already provided strong restrictions on the behavior of the Bockstein operators. Zabrodsky's theorem paved the way toward strong restrictions on the action of the remaining Steenrod operations. For example, from [12] we have the following result:

Let X be a finite H-space. Then the module of indecomposibles, Q^{2n}, is contained in the image of $\mathscr{P}^1$, provided $n \not\equiv 1 \bmod p$.

In his own work, [5], [7]–[12], and the work of R. Kane and J. Lin (who draw from Zabrodsky's work), these methods lead to a comprehensive understanding of the cohomology theory of finite H-spaces, and *a fortiori* to an understanding of the topology of Lie groups in homotopy theoretic terms.

In order to use the implication theorem, two main technical problems must be overcome. First, one usually does not have an H-map from X to the universal example E, so information about the H-deviation of $\tilde{f}$

$$D_{\tilde{f}}: X \wedge X \to E$$

is needed. In particular, one seeks to relate the H-deviation of $\tilde{f}$ to that of f. Second, one usually does not have the action of primary operations in the form $a_i x = 0$, but only modulo decomposibles. The key ideas needed to overcome these problems appear first in [10]. In essence, a filtration of H^*X is introduced to accommodate the second problem. Sufficient information to handle the first problem can then be obtained using this filtration. A theorem in [10] asserts that for any finite H-space X, the module of even dimensional indecomposibles, $\{Q^{2n}\}$, is generated over the Steenrod algebra by those in dimensions of the form

$$2\left(\frac{p^j - 1}{p - 1}\right)$$

where p is an odd prime.

In [7], Zabrodsky proves that if $x^p = py$ in the cohomology ring of a space with p-torsion free homology, then y^p is also divisible by p.

In the paper *Cohomology operations and H-spaces* [32], Zabrodsky returns to the subject to give an expository account. The theme of mutual interaction between the theories of cohomology operations and finite H-spaces is emphasized.

The main problem addressed in [5] is the structure of the cohomology ring of a finite H-space admitting a homotopy associative multiplication. In his work on Lie groups, Borel discovered that the cohomology rings of certain Lie groups are not primitively generated. Browder supplied an explanation involving both classifying spaces and infinite implications. Zabrodsky completed the picture by showing that Borel's and Browder's results depended only on homotopy associativity and not the presence of a classifying space. A key ingredient of this work is a new formula connecting H-deviations with A-deviations. The A-deviation of an H-map f between two homotopy associative H-spaces measures the obstruction to homotopy associativity of the multiplication on the homotopy theoretic fibre of f induced by a given homotopy for f.

Consider a composition of H-maps between homotopy associative H-spaces

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

and suppose $g \circ f$ is null-homotopic. Let E be the homotopy theoretic fibre of g, and $\tilde{f}$ lift f via a specific null-homotopy. Then the H-deviation of $\tilde{f}$ factors as a map

$$w : X \times X \to \Omega Z$$

and Zabrodsky's formula asserts that the A-deviation of $g \circ f$

$$\alpha : X \times X \times X \to \Omega Z$$

is given as a "boundary" of w, namely

$$\alpha(x, y, z) \simeq w(x, y) \cdot w(xy, z) \cdot w(x, yz)^{-1} \cdot w(y, z)^{-1}.$$

This formula is combined with the 1-implication formula to give several results about torsion and homotopy associativity. Among them is the theorem that if X is homotopy associative, p an odd prime and $H^*(X; Z/pZ)$ is primitively generated, then $H^*(X; Z/pZ)$ is a free algebra.

The discovery, in 1968, by P. Hilton and J. Roitberg, of H-spaces having the homotopy type of finite complexes, but not homotopy equivalent to any Lie group, S^7, RP^7 or their products, thrust H-spaces into prominence anew and uncovered new phenomena in homotopy theory. The work in his thesis put Zabrodsky in the center of activity in homotopy theory, but for many people (including this writer) the work by which Zabrodsky became first known appears in *Homotopy associativity and finite CW complexes* [6]. Here he gives the first example of a finite H-space admitting a homotopy associative multiplication but not having the homotopy type of any loop space. The construction was achieved by a novel process, now known as "Zabrodsky mixing" which has become a standard part of localization in homotopy theory. For many of us, Alex literally burst on the scene with this work, which was powerful, immediately comprehensible, and moved the subject in a sudden leap.

Most of the papers [13]–[33] are involved one way or another with the problem of classifying finite H-spaces. This problem remains open, but a substantial amount has been learned.

In the three papers [9, 11, 15], Zabrodsky addresses the following question. Write the classical Lie groups as $G(n, d)$ where $d = 1, 2, 4$ and $G(n, d) =$ SO(n), SU(n) and Sp(n) respectively. Consider the pull-back diagram

$$\begin{array}{ccc} M(n,d,\lambda) & \xrightarrow{h} & G(n,d) \\ \downarrow & & \downarrow f \\ S^{nd-1} & \xrightarrow{h_\lambda} & S^{nd-1} \end{array}$$

where the degree of h_λ is λ and f is a fibration. Then one asks, which $M(n, d, \lambda)$ are H-spaces? Zabrodsky proves that if $nd - 1$ is odd and not equal to 3, 7, then $M(n, d, \lambda)$ is an H-space if and only if λ is odd. Moreover, when λ is odd, the map h covering h_λ is an H-map. If $nd - 1$ is even, necessary conditions are given in [11].

In [16], the classification of the homotopy types among the $M(n, d, \lambda)$ is completed for $d = 2$.

In the pair of papers *The classification of H-spaces with three cells*, *I*, *II* Zabrodsky solved a vexing question concerning some of the Hilton–Roitberg examples. These can be displayed as pull-backs

$$\begin{array}{ccc} E_k & \longrightarrow & \mathrm{Sp}(2) \\ \downarrow & & \downarrow \\ S^7 & \xrightarrow{k} & S^7 \end{array}$$

where k denotes degree. Zabrodsky proves that when $k \equiv 2(4)$, E_k is not an H-space, and consequently, one can classify the simply connected finite H-spaces of rank $\leqq 2$. The proof of this result represents another side of Zabrodsky's mathematics. If necessary, he was willing and able to complete brutal calculations. The result is of significance for the wider classification problem, as this case is the first (dimension-wise) where the subtleties of the 2-local homotopy type appear.

In the papers [17, 24], Zabrodsky turned to the problem of classifying finite H-spaces after localizing at odd primes. In *On rank 2 mod odd H-spaces*, he introduces the first of his lifting theorems. He uses the new technique to prove that any H-space of rank 2 has the homotopy type of the total space of a spherical fibration over a sphere, after localization at any odd prime p where the homology is p-torsion free. Furthermore, if $\alpha \in \Pi_{q-1} S^n$ has odd order and n, q are odd, there is at most one local H-space with characteristic element α. In the paper *Torsion free mod p H-spaces of low rank* the methodology of the lifting theorem is extended to construct all p-local H-spaces with torsion free homology and having rank $< p - 1$.

The first comprehensive exposition on the lifting theorem appears in the unpublished, but widely circulated paper, *Power spaces*, written while Alex was a member of the Institute for Advanced Studies in Princeton 73/74. This work was vital for the collaboration this writer enjoyed with Alex, under the auspices of the Binational Science Foundation of Israel and the United States.

The lifting theorem was used in the construction of H-spaces with p-torsion in their homology. However, in the original work, this construction was achieved only after a tedious calculation. In the paper *Alteration of H-structures* a refinement in the technique of altering coproducts is introduced. Among the consequences is a short construction of H-spaces with torsion. In *H-spaces and self-maps* the following example is produced. For each prime $p \geqq 5$, there is a p-local H-space W of rank $p+1$ such that the mod p cohomology of W is an exterior algebra on $p+1$ classes $\mathcal{P}^i x$, $0 \leqq i \leqq p$, with $\dim x = 2p+1$. Furthermore, there is no map $f: W \to S^{2p^2+1}$ with $\deg f \equiv 1 \bmod p$.

The papers *Some relations in the mod* 3 *cohomology of H-spaces* [23] and *Evaluating a p-th order cohomology operation* [37] represent another side of the application of cohomology operations to H-spaces. In the first of these papers, a tertiary operation is used to establish the first restrictions on the action of the Steenrod algebra in the mod 3 cohomology of H-spaces with 3-torsion free homology. In the second paper, an extension of these results is made to arbitrary odd primes. These papers run parallel to the study of torsion free p-local H-spaces, and show that the theory developed in [24] cannot be extended for ranks $\geqq p$. The results in [23], [37] are also applicable to the question of which algebras over the Steenrod algebra can arise as the cohomology algebras of topological spaces. For example, taking $p=3$, then the exterior algebra

$$\Lambda(x_n, \mathcal{P}^1 x_n, \mathcal{P}^2 x_n)$$

for n odd is realizable if and only if $n \equiv 2 \bmod 3$.

The seminal examples of Hilton and Roitberg helped in revealing two new phenomena in homotopy theory. On the one hand, it is possible to have spaces X, Y which are not homotopy equivalent, but which become so after localization at any prime. On the other hand, one can have inequivalent spaces X, Y such that, after taking the product with a third space Z, $X \times Z$ and $Y \times Z$ are equivalent. To study the first situation, Mislin formulated the notion of genus. The second situation is known as non-cancellation. For H-spaces, the two phenomena were conjectured to be closely related.

Zabrodsky's contributions appear in two papers, *On the genus of finite-dimensional H-spaces* [18] and *P-equivalence and homotopy type* [19]. To avoid technical language, we shall confine attention to H-spaces, but the results in [19] apply to a significantly wider class of spaces (which are also studied in [28]).

The genus of X, denoted $G(X)$, is the set of homotopy types (of finite type) Y such that for each prime p, the p-localizations of X and Y are homotopy equivalent. Zabrodsky is able to realize every space in $G(X)$, for X an H-space, by means of the following construction. Let $K(Z, \bar{n})$ be rationally equivalent to X, and let

$$h_0 : X \to K(Z, \bar{n})$$

be any map inducing an isomorphism

$$QH^*(h_0; Z)/\text{torsion}.$$

Then every homotopy type Y in $G(X)$ appears as a pull-back

$$\begin{array}{ccc} Y & \xrightarrow{f_1} & X \\ {\scriptstyle h_1}\downarrow & & \downarrow{\scriptstyle h_0} \\ K(Z, \bar{n}) & \xrightarrow[f_0]{} & K(Z, \bar{n}) \end{array}$$

where h_1 and h_0 are $P - P_t$ equivalences and f_0, f_1 are P_t equivalences, where t is a product of primes depending only on the genus of X. Here, P is the set of all primes and P_t is the set of prime factors of t. One consequence is that $G(X)$ is a finite set. Another consequence is that if $Y \in G(X)$ then the products

$$Y^{\varphi(1/2)} \simeq X^{\varphi(1/2)}$$

are homotopy equivalent, where φ is the Euler function and t as above. Still another consequence is that

$$Y \times S \simeq X \times S$$

are homotopy equivalent, where S is a product of spheres rationally equivalent to X. The converses of these statements are true, and were first proved by Mislin and Wilkerson.

Some interesting calculations can be done. In his book Zabrodsky [2] shows that the order of $G(\mathrm{SU}_n)$ is

$$\prod_{m \leqq n} \frac{\varphi(m-1)!}{2}$$

where SU_n is the $2n$-Postnikov approximation to SU. We have

$$|G(SU(n))| \geqq |G(SU_n)|$$

but it is unknown whether this inequality is an equality. The problem bears on the relation of H-spaces with Lie groups.

Zabrodsky's book, *Hopf Spaces*, appeared in 1976. Besides his own work, he gave a careful account of virtually the entire subject of finite H-spaces, at that time. While the subject has made some significant advances since the book's appearance, a number of its topics are timely, and are not treated elsewhere. We review some of these topics.

The first two chapters (the category of H-spaces, homotopy properties of H-spaces) deal with fundamentals. A special feature is the geometric approach to the subject. In particular, the invariants known as A-deviations and C-deviations of H-maps are developed.

The third chapter (cohomology of H-spaces) develops the main results beginning with the Bockstein spectral sequence and covering much of the work reviewed in this paper. A notable feature is the geometric treatment of Browder's main results about infinite implications. This treatment does not require any analysis of chain complexes.

The fourth chapter (mod p theory of H-spaces) is alternative to theories based on localization. Contained in this chapter is the only proof in print of the fundamental theorem that a simply connected finite complex is an H-space if and only if each of its p-localizations is. The study of genus and mixing is also carried forward in this chapter.

The final chapter (non-stable BP resolutions) has the flavor of a research monograph. It advocates the use of the BP-spectrum for non-stable problems. The central construction is called "killing homology p-torsion," with the following features. Given a space X and a prime p, there exists a space $F(X)$ and a map $h: F(X) \to X$ such that (a) the integral homology of $F(X)$ is p-torsion free and (b) the homotopy groups of the fibre of h are p-torsion free, together with other properties restricting the possible homotopy types for $F(X)$ to a single mod p homotopy type. If Y has p-torsion free homology, then any $f: Y \to X$ factors through $F(X)$ with some control over choices. For example, taking $B(n, p) \doteq F(K(Z/pZ, n))$, then one has p-equivalences

$$\mathrm{SU} \underset{p}{\sim} \prod_{i=1}^{p-1} B(2i+1, p),$$

$$\mathrm{BSU} \underset{p}{\sim} \prod_{i=1}^{p-1} B(2i+2, p).$$

A development of non-stable BP-Adams resolutions is given and calculations are made. With the advances in BP-theory made since the appearance of Zabrodsky's book, one can speculate that the time is ripe to combine the insights of this chapter with the improved technology currently available.

The influence of the paper *Power spaces* has been discussed. Many of the applications depended only on rather elementary parts of this theory. Zabrodsky recognized that the basic idea was worthy of development for its own sake. His comprehensive accounts appear in two publications *Endomorphisms in the homotopy category* [26] and *On the realization of invariant subgroups of* $\pi_*(X)$ [27]. In addition there are four unpublished papers entitled *Homotopy actions I, II; the fundamental invariants*, *Polycyclic groups and the lifting theorem*, *A lifting theorem in the category of endomorphisms* and a possible 6th chapter to this paper *Groups of homotopy actions*. Announcements of these results appear as [29, 30].

Many of the applications in [26] are to the question of realizing certain algebras as cohomology rings of spaces. A notable application is the case of the polynomial algebra $Z/pZ[x]$ which is realized whenever n divides $p-1$ where $2n = \dim x$. This result was first obtained by D. Sullivan using different methods. Other results along this line appear in [21] and [28].

Perhaps the most far reaching applications of Zabrodsky's theory of self-maps to realization questions appear in [27]. As the title indicates, realization is based on invariant subgroups of homotopy rather than homology. We sketch the ideas leading to the main theorem.

Let R be an integral domain and consider sequences of polynomials P_n in $R[x]$, such that P_n divides P_{n+1}. We denote this setup by

$$P_* \in R_*[x].$$

Let φ_* be a degree 0 endomorphism of a graded R-module M. We say that P_* *annihilates* φ_* if for every m, some P_n annihilates φ_m. The product of polynomials is defined gradewise by

$$(P_* \cdot Q_*)_n = P_n \cdot Q_n.$$

Zabrodsky introduces the notion of *tensor product* of two polynomials. Essentially $P \otimes Q$ is the characteristic polynomial of $T \otimes U$ where T, U are endomorphisms of free R-modules with characteristic polynomials P, Q respectively. Then an iterated tensor product of a sequence of polynomials P_* is given gradewise by

$$(\otimes P_*)_n = \Pi P_{r_1} \otimes \cdots \otimes P_{r_t}, \qquad \Sigma r_i = n.$$

Now we can state a theorem.

Suppose X is simply connected of finite type, and T is a self-map of X. Suppose there are polynomials P_*, Q_* in $Z_*[x]$ satisfying the following three conditions:

(a) *the leading coefficient of each P_n, Q_m is prime to p,*

(b) *the* mod p *reductions of $\otimes P_*$ and Q_* are relatively prime,*

(c) *$P_* \cdot Q_*$ annihilates the induced map $\Pi_*(T) \otimes Z_p$.*

Then (in part) there exists a space Y realizing the p-local homotopy groups of X which are annihilated by products of P_*, that is,

$$\Pi_*(Y) \otimes Z_{(p)} = \lim_r \ker(P_*)^r(\Pi_*(T) \otimes Z_{(p)}).$$

Among the applications is the construction of a loop space of type (3, 7, 11, 15) whose localization at 3 is not homotopy equivalent to any Lie group, and thus is not in the genus of any Lie group. The construction of a theory to encompass this kind of realization is described in [30].

The lifting theorem is used in [26] to study the self-equivalences of a space which induce the identity in homotopy or homology. But, as remarked in [26], this approach does not yield the best results. In *Unipotency and nilpotency in homotopy equivalences* [22] methods from group theory motivate a different approach. Among the results obtained is the theorem that if X is any finite dimensional space, the group of based homotopy classes of based self-equivalences inducing the identity on $\Pi_j(X)$, $j \leqq \dim X$, is a nilpotent group.

A major result in homotopy theory in the early 1980's was the solution, by H. Miller, of a problem first posed by D. Sullivan. Zabrodsky was an active participant in the development of results based on Miller's theorem. Two papers have appeared, *On phantom maps and a theorem of H. Miller* [34] and *Maps between classification spaces* [35]. At the time of his death, Zabrodsky had in preparation at least five more papers.

The study of phantom maps in [34] is a big step in understanding this phenomenon. A map $f: X \to Y$ between spaces is called *phantom* if every composition $f \circ h$ is inessential, when $h : K \to X$ is a map from a finite complex. Prior to Zabrodsky's work examples were known, but little was known about the space of phantom maps. Under reasonable hypotheses on X, Y, Zabrodsky proves that the subspace of pointed phantom maps from X to Y is weakly equivalent to the space of all pointed maps from X to Y. This holds, for example, if X is of finite type with finite fundamental group and has only finitely many non-vanishing homotopy groups, and Y is a finite complex. Furthermore, the homotopy groups of the space of phantom maps are equal to the homotopy groups of

$$\mathrm{map}_*(X_Q, Y)$$

where X_Q is the rationalization of X in the sense of Bousfield and Kan.

In the early 1970's, D. Sullivan exhibited examples of maps between classifying spaces of compact Lie groups which were not homotopic to maps induced by homomorphisms. On the other hand, the examples suggested that homology might classify these maps. Miller's work provided access to these questions. In [35], Zabrodsky proves that if G is a compact connected Lie group with torsion free homology and H is any compact Lie group, then $f: BG \to BH$ is inessential if and only if its induced map in integral homology is trivial. He also provides an example that influenced subsequent development in the subject.

He shows that the path components of essential maps in the mapping space

$$\mathrm{map}_*(BZ/pZ, BS^3),$$

for p an odd prime, have infinitely many non-vanishing homotopy groups.

This completes our survey of Zabrodsky's papers. These contain, in addition to numerous calculations, many new and often novel constructions, both algebraic and geometric in nature. These seem likely to continue to influence the growth of mathematics in those areas where they touch. In addition to his papers, Zabrodsky made an immense contribution through conversations and correspondence. Thus he added both to knowledge and to the pleasure of working in mathematics.

BIBLIOGRAPHY OF ALEXANDER ZABRODSKY†

I. Ph.D. Dissertation

1. *On the structure of the cohomology of H-spaces*, Princeton University, 1967. Supervisors: Professors N. Steenrod and W. Browder. The dissertation in its original form was not published. An improved and generalized version was published as [5] below.

II. Book

2. *Hopf Spaces*, North-Holland Math. Studies 22, North-Holland, Amsterdam, 1976, 223 pp.

III. Publications

3. *A remark on independent functions*, Bull. Res. Counc. Isr. **9F** (1960), 135–139.
4. *Covering spaces of paracompact spaces*, Pac. J. Math. **14** (1964), 1489–1503.
5. *Implications in the cohomology of H-spaces*, III, J. Math. **14** (1970), 363–375.
6. *Homotopy associativity and finite CW complexes*, Topology **9** (1970), 121–128.
7. *On cohomology with limited torsion*, Isr. J. Math. **8** (1970), 159–164.
8. *Cohomology operations and homotopy commutativity*, Springer Lecture Notes in Math. **168** (1970), 308–317.
9. *On sphere extension of classical groups*, Symp. Pure Math. **22** (1971), 279–283.
10. *Secondary cohomology operations in the module of indecomposable*, Summer Inst. on Algebraic Topology, Aarhus 1970.
11. *On spherical classes in the cohomology of H-spaces*, Springer Lecture Notes in Math. **196** (1970), 25–33.
12. *Secondary operations in the cohomology of H-spaces*, Ill. J. Math. **15** (1971), 648–655.
13. *The classification of H-spaces with three cells. I*, Math. Scand. **30** (1972), 193–210.
14. *The classification of H-spaces with three cells. II*, Math. Scand. **30** (1972), 211–222.
15. *On the construction of new finite CW H-space*, Invent. Math. **16** (1972), 260–266.
16. *On the homotopy type of principal classical group bundles over spheres*, Isr. J. Math. **11** (1972), 315–325.
17. *On rank* 2 *mod odd H-spaces*, New Developments in Algebraic Topology, London Math. Soc. Lecture Notes #**12** (1974), 119–128.
18. *On the genus of finite dimensional H-spaces*, Comment. Math. Helv. **49** (1974), 48–64.
19. *P-equivalence and homotopy type*, Springer Lecture Notes in Math. **418** (1974), 161–171.
20. (with P. May) H^* (Sprin(n), Z_2) *as a Hopf algebra*, J. Pure Appl. Algebra **10** (1977), 193–200.
21. (with P. Hoffman) *Thin Lens Spaces*, Bull. Can. Math. **21** (1978), 31–35.
22. (with E. Dror) *Unipotency and nilpotency in homotopy equivalences*, Topology **18** (1979), 187–197.
23. *Some relations in the mod* 3 *cohomology of H-spaces*, Isr. J. Math. **33** (1979), 59–72.
24. (with G. Cooke and J. Harper) *Torsion free mod p H-spaces of low rank*, Topology **18** (1979), 349–359.
25. *Power spaces*, mimeographed.
26. *Endomorphisms in the homotopy category*, Contemp. Math. **44** (1985), 227–277.
27. *On the realization of invariant subgroups of* $\pi_*(X)$, Trans. Am. Math. Soc. **285** (1984), 467–496.

† This bibliography was prepared by Zabrodsky himself during fall 1986.

28. (with J. Aguadé) *Algebraic and geometric models for H_0-spaces*, Trans. Am. Math. Soc. **273** (1982), 181–190.

29. *On George-Cooke's theory of homotopy and topological actions*, Current Trends in Algebraic Topology, Part II, University of Western Ontario, 1982, pp. 313–320.

30. *Homotopy actions of nilpotent groups*, Contemp. Math. **12** (1982), 353–357.

31. (with J. Harper) *Alteration of H-structures*, Contemp. Math. **19** (1983), 71–84.

32. *Cohomology operations and H-spaces*, Publicacions, Seccio de Matematiques, Universitat Autonoma de Barcelona **26** (1982), 215–243.

33. (with J. Harper) *H-spaces and self-maps*, Springer Lecture Notes in Math. **1051** (1983), 371–373.

34. *On phantom maps and a theorem of H. Miller*, Isr. J. Math. **58** (1987), 129–143.

35. *Maps between classifying spaces*, in *Algebraic Topology and Algebraic K-Theory*, Ann. Math. Studies **113** (1987), 228–246.

36. (with E. Dror-Farjoun) *Homotopy equivalence of diagram of spaces*, J. Pure Appl. Algebra **41** (1986), 169–182.

37. (with J. Harper) *Evaluating a p-th order cohomology operation*, Publicacions Matemátiques **32** (1988), 61–78.

38. (with E. Dror-Farjoun) *Homotopy fixed points*, Comment. Math. Helv., to appear.

39. (with E. Dror-Farjoun) *Equivariant homotopy spectral sequence*, Springer Lecture Notes in Math. **1298** (1987), 54–81.

40. (with W. Dwyer) *Maps between classifying spaces*, Springer Lecture Notes in Math. **1298** (1987), 106–119.

41. (with W. Dwyer) *On spaces of functions between classifying spaces*, to appear.